

# IMPROVING ACCURACY AND PRECISION IN ESTIMATING FRACTAL DIMENSION OF ANIMAL MOVEMENT PATHS

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## ABSTRACT

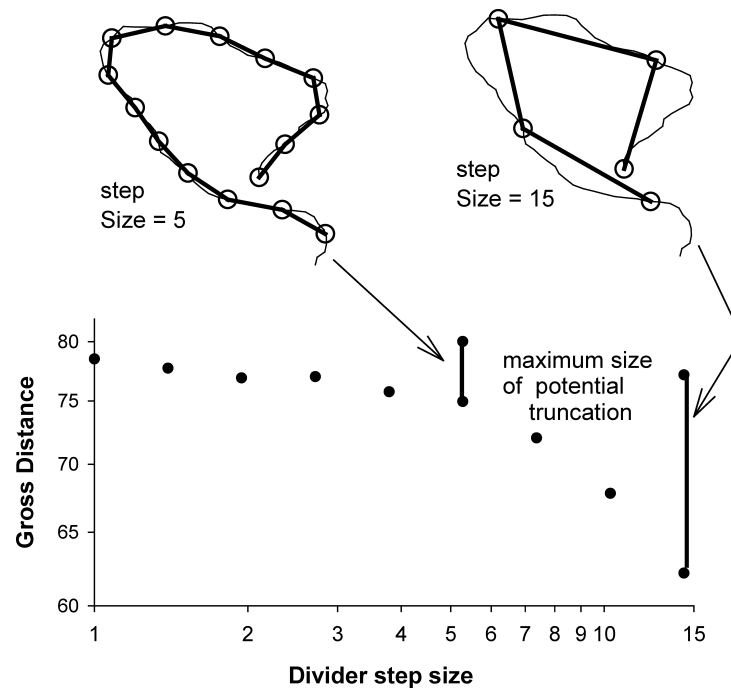
It is difficult to watch wild animals while they move, so often biologists analyse characteristics of animal movement paths. One common path characteristic used is tortuosity, measured using the fractal dimension ( $D$ ). The typical method for estimating fractal  $D$ , the divider method, is biased and imprecise. The bias occurs because the path length is truncated. I present a method for minimising the truncation error. The imprecision occurs because sometimes the divider steps land inside the bends of curves, and sometimes they miss the curves. I present three methods for minimising this variation and test the methods with simulated correlated random walks. The traditional divider method significantly overestimates fractal  $D$  when paths are short and the range of spatial scales is narrow. The best method to overcome these problems consists of walking the dividers forwards and backwards along the path, and then estimating the path length remaining at the end of the last divider step.

**Key Words:** tortuosity, bias, efficiency, divider

## 1. INTRODUCTION

The behaviour of animals while travelling affects aspects of their ecology at different spatial scales: for example, foraging behaviour (Mårell *et al.*, 2002) and animal orientation (Benhamou, 1989) at small scales, dispersal patterns at intermediate scales (Zollner and Lima, 1999), and population dynamics at large scales (With *et al.*, 2002). However it is difficult to study behavioural aspects of these directly because it is hard to watch wild animals while they are moving, especially at large distances and over long time intervals. It is much easier to record animal locations. Thus as a proxy to watching moving animals we often analyse various characteristics of their movement paths.

One common movement path characteristic used is tortuosity, measured using the fractal dimension ( $D$ ). Fractal dimension is the continuous analogue of discrete geometric dimensions (Mandelbrot, 1967; Milne, 1991); for example, lines have a geometric dimension of 1 and planes a dimension of 2. The fractal dimension for movement paths lies between 1 and 2 - i.e.  $D = 1$  when the path is straight and  $D = 2$  when the path is so tortuous that over time it will completely cover a plane. Fractal analysis has been used in various types of studies of animal movement – ranging from the landscape perceptions of grasshoppers (With, 1994), habitat selection at different spatial scales of marten (Nams and Bourgeois, 2004), to scale-dependent movements of seabirds (Fritz *et al.*,



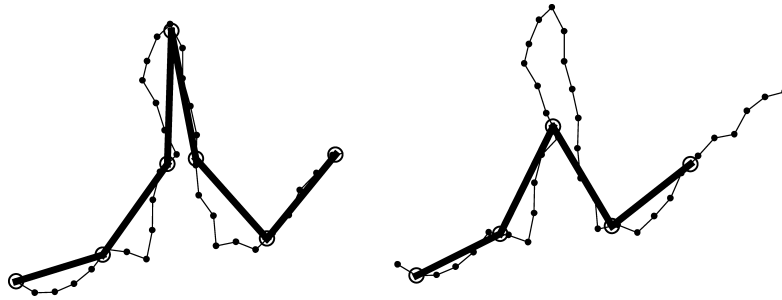
**Figure 1.** Example of path truncation. Gross distance is estimated for this path with dividers of various step sizes. At divider sizes 5 and 15 the gross distance is underestimated by up to 5 and 15 units. Since  $D$  is based on the slope of the plot, this truncation could have major effects

2003). Fractal analysis is especially useful when studying how animals change in their response to their environment with changes in spatial scale (Nams, 2005).

Typically, biologists estimate fractal  $D$  of animal paths by using the dividers method (Mandelbrot, 1967; Milne, 1991). The length of the path is measured by walking a pair of dividers of a certain size along the path. If this is done for larger and larger dividers, then the slope of the plot for  $\log(\text{path length})$  vs  $\log(\text{step size})$  is  $1-D$ , yielding one overall estimate for  $D$  over a range of scales (Mandelbrot, 1967).

There are two potential problems with the divider technique for estimating fractal dimension. The first one affects accuracy (i.e. the closeness to the true value). The last divider step rarely falls directly on the end of the path; if path length is estimated using discrete units of divider steps then path length is underestimated – it is truncated (Figure 1). The underestimate would be greater at larger scales, leading to a steeper relationship between  $\log(\text{path length})$  vs  $\log(\text{divider size})$  and thus an overestimate of  $D$ . No one has discussed how to deal with the truncation problem, and it is not known how large a bias this would create.

The other problem affects precision (i.e. repeatability). As the dividers walk along the path, at some locations they may land right on the middle of the bend of a curve, and at other locations they may bypass over a curve. Whether they land in a curve depends



**Figure 2.** Example of sensitivity of gross distance to starting point. When the dividers are started at the first point then one divider step happens to land in the most extreme part of the curve. When the dividers are started on step 2, they miss the major part of the curve – thus changing the estimate of gross distance

on the starting point. Thus the estimated gross distance is sensitive to the starting point of the dividers (Figure 2), increasing potential variation in  $D$ . It has been suggested that this problem be addressed by using several different starting points, and averaging the resulting  $D$  values (With, 1994). However this method has not been tested, nor is it known how much this problem would affect precision.

In this paper I measure the bias caused by the truncation problem and test a method for minimising the problem. I also test three methods for minimising the variation due to divider starting point.

## 2. ALTERNATE ESTIMATES OF $D$

### Accuracy

I adjusted for the truncation error by adding the straight-line distance between the end of the last divider and the end of the path. This effectively estimates the distance of the last segment, but might still underestimate path length when movement paths are very tortuous and with large divider sizes. This will be called the adjusted  $D$ .

### Precision

I tested four methods.

- (1) The original divider estimate, with the dividers traversing the path once.
- (2) This method calculates  $D$  two times, once going forwards along the path and starting at the first point, and once going backwards and starting at the last point. The mean of these two  $D$ -values is then used. No data points are dropped from the analysis, but the disadvantage is that only two different starting points are used. I will call this estimate  $D_{\text{Mean}}$ .
- (3) This method calculates  $D$  for different starting points near the start of the path, dropping the first previous steps from the analysis. The mean  $D$  from all iterations is used (With, 1994). As suggested by With (1994), I tested this using the first four steps of the movement path. An advantage is that this method uses different starting points, but a disadvantage is that the initial points are dropped from the

analysis thus reducing the sample size and as a consequence, statistical power. I will call this estimate  $D_4$ .

- (4) The final method estimates gross path length many times. Each time the method starts the dividers on a randomly-chosen step on the path and walks the dividers both forwards and backwards. Then it uses the mean of these distance estimates. I tested this with 20 resamples. An advantage is that this method samples the path many times, but a disadvantage is that the truncation problem occurs at both ends for each iteration. I will call this estimate  $D$  Many.

For each of these methods I used the adjusted  $D$  method of dealing with path truncation.<sup>1</sup>

### 3. SIMULATIONS

#### Methods

I tested the various methods of fractal  $D$  estimation by applying them to simulated correlated random walks. The correlated random walk has been proposed as a null model for animal movements in general (Turchin, 1996), and has been shown to fit a variety of animals (Cain, 1985; Benhamou, 1990; Wallin, 1991). With a correlated random walk, an animal moves with discrete steps, and at each step the animal turns a certain random angle from the direction of the previous step. I modelled a simple correlated random walk, with fixed step lengths (e.g. Boone and Hunter, 1996), and turning angles distributed with a circular normal distribution (Cain, 1985).

I varied four factors that might affect this error: the divider step sizes, path length, the range of divider step sizes, and path tortuosity. A larger divider size would increase the maximum truncation error and a shorter path would increase truncation error relative to gross distance. Both a more tortuous path and a wider range of divider step sizes would increase the difference in truncation error among small to large divider sizes.

I varied path tortuosity by changing the parameter for turning angle concentration,  $K$ , where larger  $K$ -values represent straighter movement paths (Cain, 1985). I simulated paths of two tortuosities by using two different  $K$ -values: a tortuous path with  $K = 2$  and a more linear path with  $K = 10$  (Figure 3).

I created paths of different lengths by simulating a 10000-step path for each  $K$ -value and dividing it into equal sections of the desired path length. Path lengths of 80, 100, 150, 250, 500 and 1000 steps were used, resulting in 125, 100, 66, 40, 20, and 10 paths of each length, respectively. Using the same initial movement path for all subdivided path lengths ensured that comparisons among path lengths did not include variation due to the random nature of the generation of the movement paths.

I varied divider step sizes by using two sets of sampling intervals for each type of movement path. For the tortuous paths ( $K = 2$ ) the intervals ranged from 1 to 10, and from 2.5 to 10 units, divided into 10 segments. For the straight paths ( $K = 10$ ) 1–20, and 5–20 were used, divided into 10 segments. These combinations of ranges of intervals and path lengths gave a range of number of divider steps per path from 2 to 1000 (Table I).

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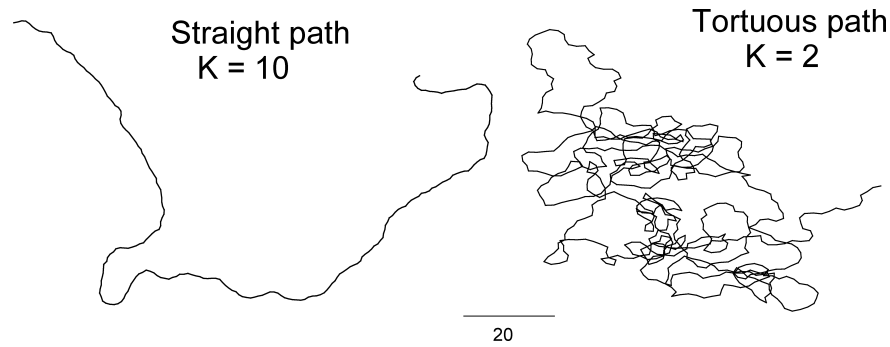
<sup>1</sup>A computer program to carry out these estimates can be downloaded from [www.nsc.ca/envsci/staff/vnams/](http://www.nsc.ca/envsci/staff/vnams/)

**Table 1.** Ranges of numbers of divider steps used in analysis of simulated movement paths

Path length (steps)	Number of paths	Straight paths, $K = 2$				Tortuous paths, $K = 10$			
		Narrow range: 1–20		Wide range: 5–20		Narrow range: 1–10		Wide range: 2.5–10	
		Min $n^a$	Max $n^b$	Min $n^a$	Max $n^b$	Min $n^a$	Max $n^b$	Min $n^a$	Max $n^b$
80	126	14	2	78	2	24	2	74	2
100	101	18	3	98	3	30	3	93	3
150	67	27	5	147	5	46	5	140	5
250	40	46	8	247	8	76	9	235	9
500	20	94	18	500	18	154	18	471	18
1000	10	189	36	1000	36	308	39	944	39

<sup>a</sup>Min  $n$  represents the number of divider steps at the smallest scale of the range

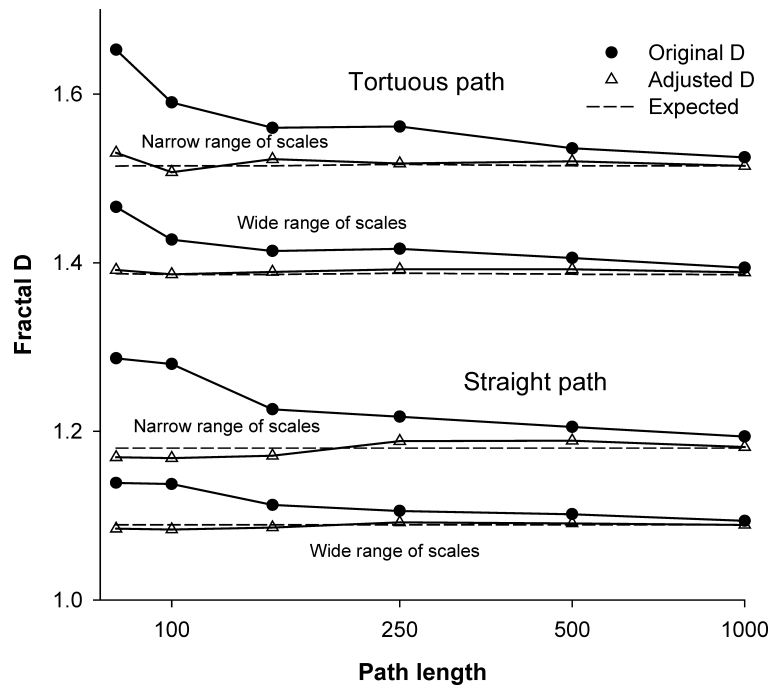
<sup>b</sup>Max  $n$  represents the number of divider steps at the largest scale of the range


**Figure 3.** Examples of the two types of movements paths used in the simulations. Both paths displayed are 250 steps long

I estimated the fractal  $D$  for each movement path, and then averaged those over all paths of that length. Precision was estimated by the standard deviation of each  $D$  estimate. The whole 10000-step path was used to estimate a less biased estimate of  $D$  (referred to as the “expected  $D$ ”) for a given correlated random walk and used to compare with various shorter, subdivided pathways.

#### 4. RESULTS

The adjusted  $D$  was more accurate than the original  $D$  for all movement paths. The original  $D$  overestimated fractal  $D$  more for shorter paths at each path length and sampling scale interval (Figure 4). The bias ranged from a negligible amount at path lengths of 1000 to a maximum of 0.12 at a path length of 80. Bias increased with shorter paths,



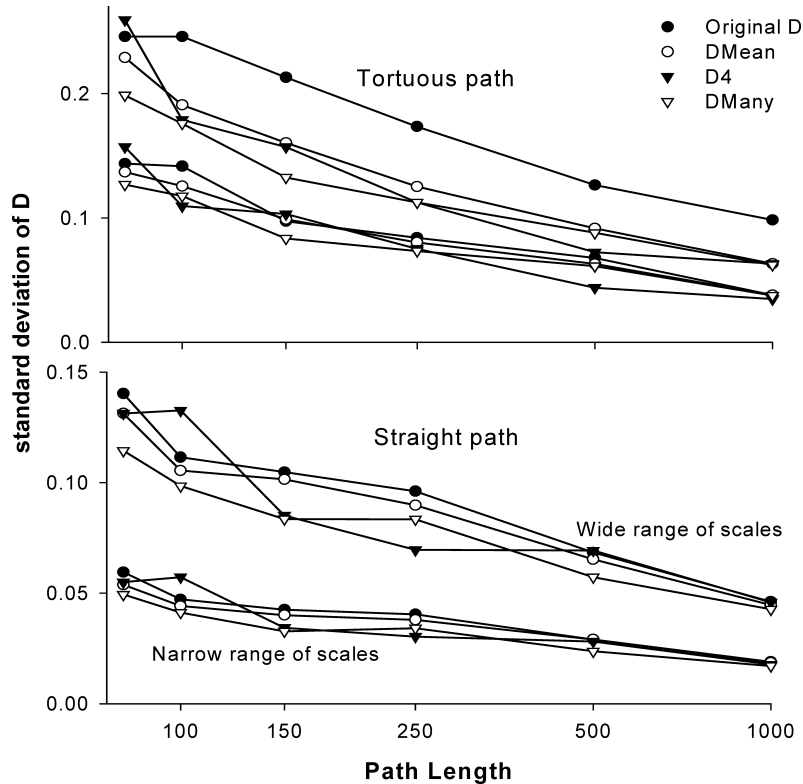
**Figure 4.** Accuracy of truncation estimate. The original and adjusted  $D$  estimators were applied to movements paths of varying lengths and tortuosities, sampled at two different ranges of spatial scales. The dotted line represents an “expected”  $D$ , estimated using the whole 10000-step movement path. The adjusted  $D$  estimator was more accurate at all combinations of parameters

more tortuous paths, and sampling over a shorter range of spatial scales. The adjusted  $D$  had a maximum bias of only 0.01 at a path length of 80.

Of the four precision estimates tested, the original  $D$  gave the poorest precision in almost all situations (Figure 5), except for a few when the  $D4$  was slightly less precise. The  $D4$  varied the most in precision, giving the lowest at a path length of 100 for the straight paths, yet the highest for a path length of 250 for the straight path. Perhaps this happened because of the deletion of the first few steps for part of the analysis, where the variation depended on which specific steps were deleted. The  $DMany$  gave the highest precision in most cases, although the difference was quite small.

Although the  $DMany$  was most precise it was not the most accurate (Figure 6).  $DMany$  underestimated fractal  $D$  when paths were short, especially for tortuous paths. The other three estimators were accurate for most simulations.

Considering both precision and accuracy, the best estimate is to combine  $DMean$  and the adjusted  $D$ . However, the range of divider sizes should be given careful consideration. The adjusted  $D$  can still be biased at larger divider sizes. The original  $D$  is inaccurate. While the  $D4$  is similar in precision as the  $DMean$ , it varies in precision. While the  $DMany$  is the most precise, it is the least accurate. The  $DMean$  and adjusted  $D$  combine accuracy with low precision at all combinations of movement path parameters.

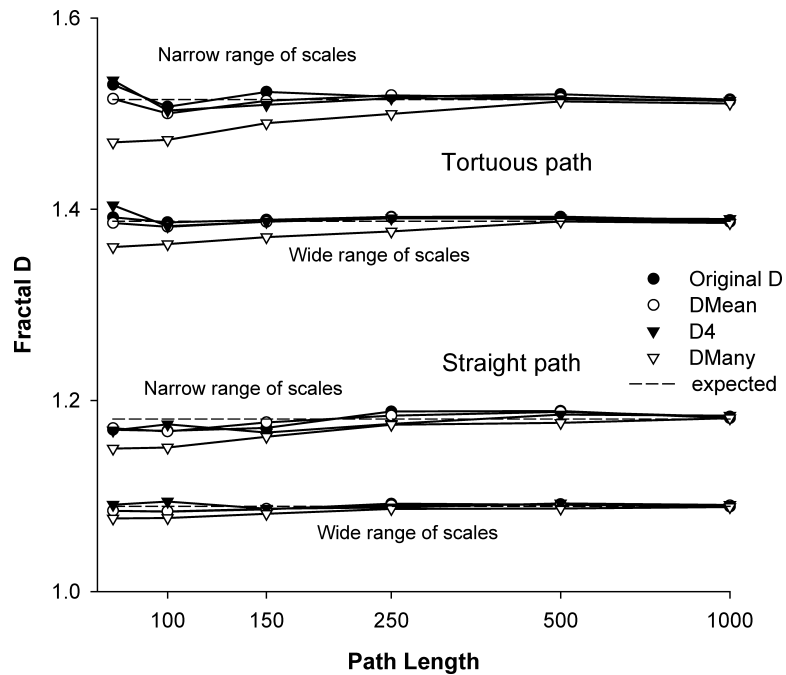


**Figure 5.** Precision of resampling estimates. The four estimators were applied to movements paths of varying lengths and tortuosities, sampled at two different ranges of spatial scales. A small standard deviation is more precise. The original  $D$  was least precise at all combinations of parameters

## 5. EXAMPLE: SMALL MAMMAL MOVEMENT PATHS

Red-backed voles (*Clethrionomys gapperi*) are small rodents that live in the boreal forest, with home ranges of about  $50\text{ m} \times 50\text{ m}$  (Merritt, 1981). They live on the forest floor and in burrows just under the forest floor. They are the main base for the predatory food chain in the boreal forest. I gathered data from movement paths from red-backed voles near Truro, Nova Scotia, Canada (Nams, 2005). The animals were tracked by following a tiny thread spool attached to their backs (as in Boonstra and Craine, 1986). As the thread plays out, it snags in grass and other vegetation on the forest floor, thus retaining the shape of the movement path. The trails were mapped by first laying a thick bright rope on the thread trail, then video-taping the rope and digitising the images. This gave  $x, y$  co-ordinates for the trails with an accuracy of  $\pm 5\text{ cm}$ . Sixty eight movement paths with lengths ranging from 2.30 to 133.2 m, and number of data points ranging from 112 to 8773, were obtained.

A fractal  $D$  was estimated using the DMean and original  $D$  estimators for each vole movement path, with same setting of the range of divider step sizes for each path (5 to



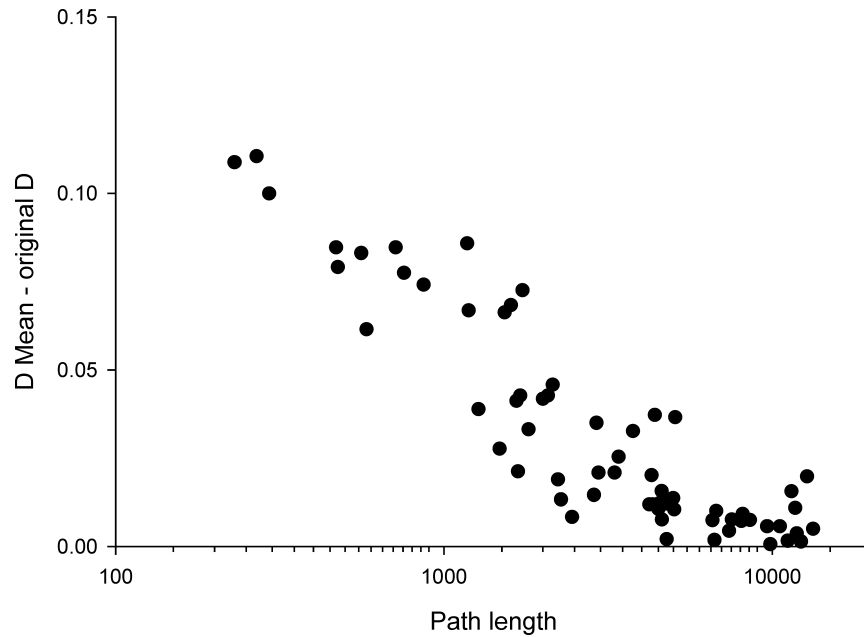
**Figure 6.** Accuracy of resampling estimates. The four estimators were applied to movements paths of varying lengths and tortuosities, sampled at two different ranges of spatial scales. The dotted line represents an “expected” fractal  $D$ , estimated using the whole 10000-step movement path.  $D$  Many was least accurate, while the rest of the estimators were similar in accuracy

500 cm, with 20 equidistant divisions (log)). In addition, fractal  $D$  was estimated as a function of spatial scale. To measure changes in  $D$  with scale using the divider method, one measures  $D$  over a narrow window range in scale, and repeats this for different window ranges (Krummer *et al.*, 1987; Nams, 1996; Nams and Bourgeois, 2004). The means and confidence intervals of these estimates were then estimated over all movement paths.

The original  $D$  estimate consistently overestimated overall Fractal  $D$ , as compared to the  $D$  Mean, with the bias increasing with decreasing path length (Figure 7). When estimating  $D$  vs spatial scale, the original  $D$  estimate overestimated fractal  $D$  at larger spatial scales (Figure 8).

## 6. DISCUSSION

Are these biases large enough to be biologically important? Yes. First, a small difference in  $D$  can represent a large biological difference, because  $D$  represents an exponent in scaling relationships, and a small change in an exponent can have a big impact on estimates of distance travelled (Milne, 1997). Second, past studies of  $D$  of movement paths have found significant differences among movement paths similar in size to the



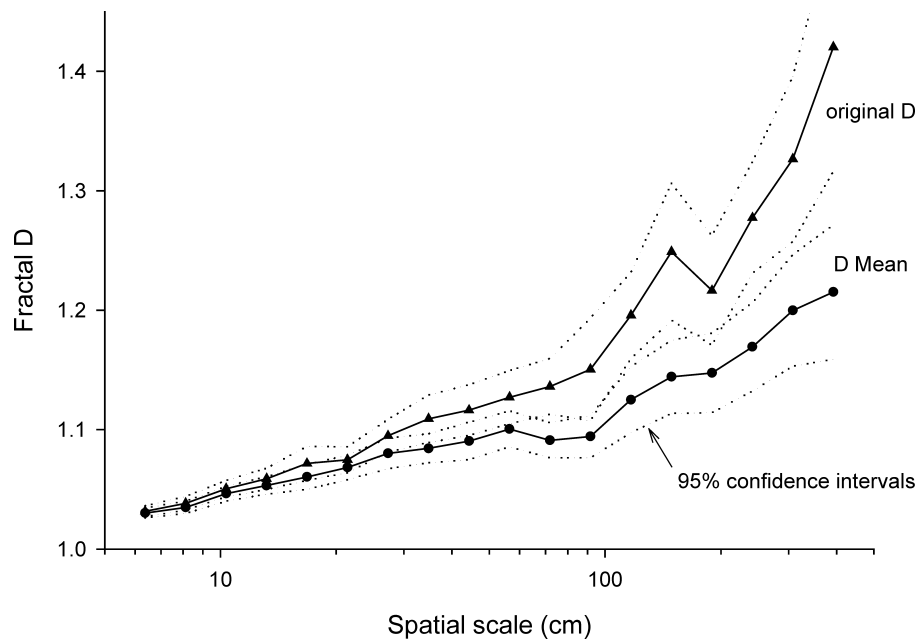
**Figure 7.** Difference between estimates of the original  $D$  and  $D$  Mean, as a function of path length. Each data point represents one vole movement path

biases I found. The bias in fractal  $D$  could be as large as 0.12. Fauvergue *et al.*, (1998) found significant differences in fractal  $D$  of 0.13 between movement paths of species of male parasitoid wasps responding to female sex pheromones, and With (1994) found significant differences in fractal  $D$  of 0.09 between adult vs nymph *Opeia obscura* grasshoppers. A bias of 0.12 might significantly affect such results.

Even more important than the size of the bias, is that the bias is not constant – it varies with path tortuosity, path length (Figure 7) and the scale of sampling (Figure 8). It is difficult to keep these parameters constant, especially when gathering data from wild animals. For example Phillips *et al.*, (2004) tracked foraging red fox and striped skunks, getting varying path lengths, from  $n = 10$  to 40.

This points out future research. The biases in my simulations were predictable, using parameters that were constant and well behaved; but field data are highly variable and we do not know how the contrasting estimators will perform. It would be useful to study simulations that examine the influence of a range of model parameters (i.e., “field variation”) on the biases in the contrasting estimators. A comparison between well behaved simulations and simulations with noisy parameters may give us an idea of the degree and source of biases in the estimators using field data. It would be very useful if the biases could be measured and identified under these conditions.

In conclusion, the inherent bias and variability in the traditional divider method for estimating fractal  $D$  can be minimised by a combination of adjusting for path truncation



**Figure 8.** Comparison of two estimators for  $D$  vs scale. At each scale, a window range of  $\pm 0.25$  (on a log scale) was used to estimate fractal  $D$ . The original divider estimate consistently overestimates  $D$  as compared to  $D$  Mean. Each data point is a mean of values from all 68 vole movement paths

at the end of the path, and running the iteration twice, forwards and backwards along the movement path. There are no disadvantages to using these estimates.

### ACKNOWLEDGEMENTS

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